

A Note on Construction of Asymmetrical Main Effect Plans

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SUMMARY

A new series of orthogonal main effect plans for $4^{n_1} \times 3^{n_2} \times 2^{n_3}$ in 49 runs is obtained.

Key words : Asymmetrical orthogonal main-effect plan; Regular group divisible design.

Introduction

Major work on orthogonal main effect plans for asymmetrical factorials has been done by Addelman [1]; Addelman and Kempthorne [2]; Margolin [9]; Starks [3]; Dey and others [7], [8], [3] Chacko et al [4] [5]. Balanced arrays (B-arrays) have been found to be extremely useful in constructing balanced factorial designs. Sinha and Nigam [11], and Nigam [10] constructed a series of $(n+1)$ symbol B-arrays of strength two from regular group divisible designs $V = mn; b, r, k, m, n, \lambda_1 = 0, \lambda_2 > 0$. Using these B-arrays with $V = mn$ Sinha and Nigam [11] have constructed some orthogonal main effect plans.

In this note, a new series $4^{n_1} \times 3^{n_2} \times 2^{n_3} / 49$ of asymmetrical orthogonal main effect plans is constructed, using the incidence matrix of a regular group divisible design reported by De and Roy [6] following the procedure by Sinha and Nigam [11] as possibly existent regular group divisible design.

2. Construction

First the incidence matrix of the design with parameter $v = b = mn, r, k, m, n, \lambda_1 = 0$ and $\lambda_2 > 0$ is written using 1 or -1 as its elements. Next it is modified by including p more rows of the type $(-1, -1, \dots, -1)$ where $p = (r^2 - b\lambda_2) / \lambda_2$ has to be a positive integer other than zero.

The columns of this modified incidence matrix are grouped by placing consecutively the columns corresponding to the treatment in each group to form m submatrices of order $\{(mn + p) \times n\}$. The $\{(mn + p) \times n\}$ submatrix consists of rows of the form.

	1	2	n
1.	-1	-1	-1
2.	1	-1	-1
3.	-1	1	-1
.....				
.....				
n+1	-1	-1		1

The above treatment combinations occur equal or proportional number of times in $(mn + p)$ rows in each submatrix. We can replace the above treatment combinations with the levels $0, 1, 2, \dots, n$ of a factor by replacing the rows of m submatrix by the above levels. We get symmetrical orthogonal main effects plan of the type $(n + 1)^m$ in $(mn + p)$ runs studied by Nigam [10]. Now the existence of G.D. design with parameters $v = b = 45, r = k = 7, m = 15, n = 3, \lambda_1 = 0, \lambda_2 = 1$ has been reported by De and Roy [6]. Thus construction of MEOP of the type 4^{15} in 49 runs has become possible as conjectured by Nigam [10]. Hence reduced MEOP of the type 3^{15} and $4^{n_1} \times 3^{n_2} \times 2^{n_3}, \Sigma n_i = 15$ in 49 runs, can also be obtained as described below.

As we know that the row of each submatrix consists of $(n + 1)$ distinct treatments combination of n factors each at level -1 or $+1$. Now we replace $(n + 1)$ distinct row of n columns of each n_1 submatrices with the levels $0, 1, 2, \dots, n$; n distinct rows of $(n - 1)$ columns of each n_2 submatrices with the levels $0, 1, 2, \dots, (n - 1)$; $(n - 1)$ distinct rows of $(n - 2)$ columns of each n_3 submatrices with the level $0, 1, 2, \dots, (n - 2)$ and so on upto rows of single column of each n_m sub-matrices with the levels of 0 and 1 . Thus we get orthogonal asymmetrical main effect plan of the following type.

$$(n + 1)^{n_1} \times n^{n_2} \times (n - 1)^{n_3} \times \dots \times 2^{n_m}, \text{ where } \Sigma n_i = m$$

Thus utilizing incidence matrix of RGD with parameters $v = b = 45, r = k = 7, m = 15, n = 3, \lambda_1 = 0, \lambda_2 = 1$, after adding four rows of -1 's, we may obtain MEOP of the type 3^{15} and where $4^{n_1} \times 3^{n_2} \times 2^{n_3}/49$, where $\Sigma n_i = 15$ by the above method. These plans are not reported earlier.

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A Class of Unbiased Dual to Ratio Estimator in Stratified Sampling

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SUMMARY

This paper proposes a class of unbiased dual to ratio estimator for population mean and analyses its properties.

Key words : Dual to ratio estimator, Positively correlated auxiliary variables, Optimum estimator, Variance.

Introduction

Assume that the population of size N is divided into L strata and that sampling within each stratum is simple random sampling without replacement (SRSWOR). Let N_h denote the number of units in the h -th stratum and n_h the size of the sample to be selected therefrom, so that

$$\sum_{h=1}^L N_h = N \text{ and } \sum_{h=1}^L n_h = n.$$

$$\text{Let } \bar{Y} = \frac{1}{N} \sum_{h=1}^L \sum_{j=1}^{N_h} y_{hj} = \frac{1}{N} \sum_{h=1}^L N_h \bar{Y}_h = \sum_{h=1}^L P_h \bar{Y}_h, \text{ and}$$

$$\bar{X} = \frac{1}{N} \sum_{h=1}^L \sum_{j=1}^{N_h} x_{hj} = \frac{1}{N} \sum_{h=1}^L N_h \bar{X}_h = \sum_{h=1}^L P_h \bar{X}_h$$

be the population means of positively correlated characters y (study) and x (auxiliary) respectively, where $P_h = N_h/N$, $\bar{Y}_h = \sum_{j=1}^{N_h} y_{hj}/N_h$ and

$\bar{X}_h = \sum_{j=1}^{N_h} x_{hj}/N_h$. To estimate \bar{Y} , the usual separate ratio estimator is defined by

$$\hat{\bar{Y}}_{RS} = \sum_{h=1}^L P_h \bar{y}_{Rh} \tag{1.1}$$

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